IPM/P-2005/039

On the Anomalies and Schwinger Terms in Noncommutative Gauge Theories

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Abstract

Invariant (nonplanar) anomaly of noncommutative QED is reexamined. It is found that just as in ordinary gauge theory UV regularization is needed to discover anomalies, in noncommutative case, in addition, an IR regularization is also required to exhibit existence of invariant anomaly. Thus resolving the controversy in the value of invariant anomaly, an expression for the unintegrated anomaly is found. Schwinger terms of the current algebra of the theory are derived.

PACS No.: 11.15.-q, 11.30.-j, 11.30.Rd, 11.40.-q, 11.40.Ex, 11.40.Ha

Keywords: Noncommutative Field Theory, Nonplanar Anomaly, Schwinger Term

1 Introduction

One of the greatest achievements of theoretical physics in the second half of the last century is the discovery of the breakdown of classical symmetries of field theories due to quantum effects [1, 2, 3, 4]. The far reaching consequences of this discovery include quantitative predictions of physical amplitudes from anomaly in global symmetries such as in two photon decay of pions, and restriction of consistent gauge theory models of particle physics from cancellation of anomalies in local symmetries such as in electroweak theory (for recent review of anomaly see [5] and references therein).

Therefore, in study of various field theories, it is essential to understand anomalies of its various classical global and local symmetries. One class of field theories studied extensively recently is noncommutative field theories, in particular gauge theories. Anomalies of noncommutative gauge theories have been widely studied [6, 7, 8, 9, 10, 11] and are mostly well understood.

In ordinary commutative gauge theories, currents corresponding to *global symmetries* satisfy an ordinary divergence equation

$$\partial_\mu j_5^\mu = 0,$$

while chiral currents corresponding to *local symmetries* satisfy a covariant divergence equation

$$D_\mu J^\mu = 0,$$

where $D_\mu = \partial_\mu - igA_\mu$, with A_μ acting in the adjoint representation of the gauge group. Generally these two divergence equations receive nonzero contributions from quantum correction.

In the noncommutative gauge theories a similar situation exists. For simplicity we restrict our attention to a $U(1)$ noncommutative gauge theory, as noncommutativity already incorporates complications inherent in the non-Abelian nature of gauge theories with larger gauge groups.

In this case, there are two distinct global Noether currents which play the role of the two currents above, the invariant $j_\mu^5 \equiv \bar{\psi}_\alpha \star \psi_\beta (\gamma_\mu\gamma_5)^{\alpha\beta}$, and the covariant current $J_\mu^5 \equiv \psi_\beta \star \bar{\psi}_\alpha (\gamma_\mu\gamma_5)^{\alpha\beta}$. These two currents correspond to the same global $U(1)$ symmetry and have divergence equations

$$\partial_\mu j_5^\mu = 0, \tag{1.1}$$

and

$$D_\mu J_5^\mu = 0, \tag{1.2}$$

where D_μ acts in the $\star$-adjoint operation to be defined in section 2.

The anomaly of the covariant conservation equation (1.2) has been unequivocally calculated using various UV regularization methods in [6, 8, 11, 12, 13] and is

$$D_\mu J_5^\mu(x) = -\frac{g^2}{16\pi^2} F_{\mu\nu}(x) \star \tilde{F}^{\mu\nu}(x). \tag{1.3}$$

The anomaly contribution to the invariant current j_5^μ is however less uncontroversial [7, 9, 10, 13].

In fact anomaly of $\partial_\mu j_5^\mu$ was calculated with a UV cutoff in the spirit of UV/IR mixing phenomenon [14] in [7] and found to be

$$\partial_\mu j_5^\mu(x) = -\frac{g^2}{16\pi^2} F_{\mu\nu}(x) \star' \tilde{F}^{\mu\nu}(x) + \cdots, \tag{1.4}$$

for the limit of small noncommutative momentum relative to the UV cutoff; while it was observed in [8] that the contribution to this conservation equation involve only nonplanar diagrams which are convergent and therefore need no UV regularization, leading to zero anomaly.

This result was confirmed in [9] on the basis of string theoretical consideration and suggested to be the consequence of Green-Schwarz mechanism operating autonomously in field theory. Later authors of [10] pointed out that as the two currents J_5^μ and j_5^μ lead to the same global symmetry and charge

$$Q_5 = \int d^3x J_5^0(x) = \int d^3x j_5^0(x), \tag{1.5}$$

and as J_5^μ has an anomaly, then j_5^μ must be also anomalous and in fact

$$\int d^2x_{NC}\; D_\mu J_5^\mu(x) = \int d^2x_{NC}\; \partial_\mu j_5^\mu(x). \tag{1.6}$$

These authors proposed to resolve this paradox by quoting Ref. [7] that in the momentum representation, after taking the infinite limit of the UV cutoff, anomaly is zero everywhere except at the zero momentum in the noncommutative directions. They made an independent calculation in the point-splitting method and confirmed that their zero momentum point anomaly is finite. Yet, close scrutiny of the calculation reveals that the anomaly in the coordinate representation gives zero as it is zero everywhere in the momentum representation except for a finite value at zero momentum, this having measure zero in the Lebesgues measure.

Thus the paradox remains. The authors of Ref. [10] realize this difficulty where they take the infinitesimal parameter of gauge transformation in the variation of the effective action, to be a Dirac δ-function in momentum variables.

Our proposal is that just as in the ordinary commutative theory anomaly will not appear unless a UV regularization[1] is introduced and careless manipulation of divergent integrals are avoided, so it is in the noncommutative gauge theory: In addition to the UV regularization, an IR regularization must also be introduced to exhibit a finite anomaly in the invariant current divergence $\partial_\mu j_5^\mu$, thus resolving the above paradox. For this regularization we use compactification of the noncommutative directions[2] with radius R. We find an unintegrated form of anomaly for finite R

$$\partial_\mu j_5^\mu = -\frac{1}{(2R)^2}\,\frac{g^2}{16\pi^2}\int_{-R}^{+R} d^2x_{NC} F_{\mu\nu}\tilde{F}^{\mu\nu}.$$

Integrating both sides over noncommutative directions, the R dependence on the r.h.s. cancels out, satisfying the constraint equation (1.6) for $R \to \infty$ limit. The resulting integrated form of the anomaly is in accordance with Ref. [10]. of Armoni, Lopez and Theisen. On the other hand, taking the decompactification limit first and then integrating over noncommutative directions yields a zero anomaly. In agreement with Ref. [9] of Intriligator and Kumar.

We will then use the same technique to calculate the current commutators and obtain Schwinger terms reminiscent of the central charge of the affine algebra appearing in conformal field theories.

There has been another calculation of the invariant anomaly [12] which gives an unintegrated result on which we will briefly comment later.

In the next section, we set down our notation and briefly review the various calculations of axial anomalies for the commutative and noncommutative gauge theories. In section 3, we make careful point-splitting calculation in the presence of both a UV and an IR regulators.

In section 4, we present our results for current algebra of the theory and section 5 is devoted to discussion where string theoretical aspects are touched upon.

2 Background on Anomalies

The simplest and earliest and the most thoroughly studied classical symmetry broken by quantum corrections is the axial symmetry

$$\psi(x) \to e^{i\gamma_5\alpha}\psi(x), \tag{2.1}$$

[1]It is sometimes possible to obtain anomalies as IR phenomenon [15].

[2]Similar compactification in other contexts of noncommutative field theory were performed in [16].

of massless QED

$$\mathcal{L} = \bar{\psi}\left(i\not{\partial} - g\not{A}\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{2.2}$$

with its classical current conservation

$$\partial_\mu j_5^\mu = 0, \qquad\qquad j_5^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi, \tag{2.3}$$

broken by the anomaly [1]

$$\langle\partial_\mu j_5^\mu\rangle = -\frac{g^2}{16\pi^2}F\wedge F \equiv -\frac{g^2}{16\pi^2}\varepsilon_{\mu\nu\lambda\nu}F^{\mu\nu}F^{\lambda\nu}. \tag{2.4}$$

This effect was first discovered through careful analysis of the divergent triangle diagram contributions to the vacuum expectation value of the current

$$\langle\partial_\mu j_5^\mu(x)\rangle = \int d^4y\; d^4z\; \partial_\mu\Gamma^{\mu\lambda\nu}(x,y,z)A_\lambda(y)A_\nu(z), \tag{2.5}$$

in which the divergence of the three-point function $\Gamma^{\mu\lambda\nu}$ is the sum of two divergent integrals of the cross diagrams (see Figure 1)

$$\begin{aligned}\partial_\mu\Gamma^{\mu\lambda\nu}(x,y,z) \equiv \partial_x^\mu\langle\mathrm{T}(j_5^\mu(x)j^\lambda(y)j^\nu(z))\rangle = -g^2\int\frac{d^4k}{(2\pi)^4}\,\frac{d^4p}{(2\pi)^4}\; e^{-i(k+p)x}e^{iky}e^{ipz}\\ \times\int\frac{d^4\ell}{(2\pi)^4}\;\mathrm{tr}\left(\gamma^\mu\gamma^5\frac{(\not{\ell}-\not{k})}{(\ell-k)^2}\gamma^\lambda\frac{\not{\ell}}{\ell^2}\gamma^\nu\frac{(\ell+\not{p})}{(\not{\ell}+p)^2}\right) + (\lambda\leftrightarrow\nu, k\leftrightarrow p)\,,\end{aligned} \tag{2.6}$$

which would cancel to give zero anomaly if a simple shift of integration is performed.

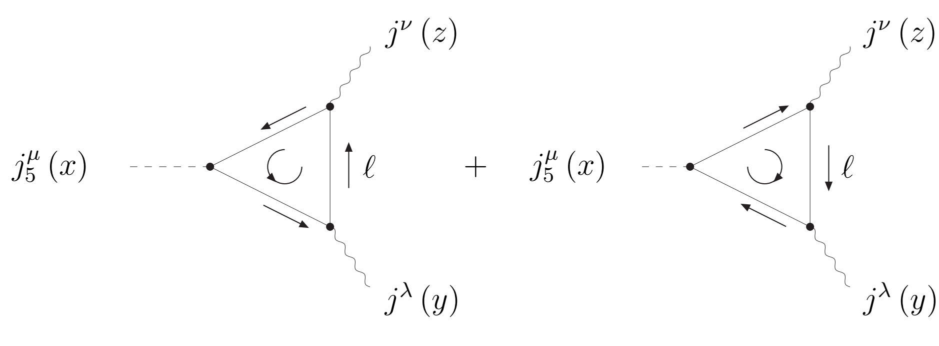

Figure 1: Triangle diagrams giving rise to the axial anomaly.

However, as the integrals are linearly divergent such change of variable may not be admissible. After UV regularization of the integrals the nontrivial anomaly of (2.4) arises.

This same result is obtained by another UV regularization of point-splitting of the poorly defined operator product $\bar{\psi}\psi$

$$j_5^\mu(x) = \lim_{\epsilon\to 0} \bar{\psi}(x+\frac{\epsilon}{2})\gamma^\mu\gamma^5 \;\exp\left(ig\int\limits_{x-\frac{\epsilon}{2}}^{x+\frac{\epsilon}{2}} A_\mu dx^\mu \right)\psi(x-\frac{\epsilon}{2}). \tag{2.7}$$

A more elegant but less rigorous calculation of anomaly was obtained by the observation that in the path integral calculation of the vacuum expectation value of the axial vector current, the measure of the path integral of the partition function is not invariant under the map (2.1); therefore the divergence of the current j_5^μ obtains a quantum correction which may be calculated with a UV regularization of the path integral measure Jacobian [17].

The method we use in this work, the point-splitting method, will be dealt with in detail for a noncommutative theory in section 3.

Generalization of the anomaly phenomenon to non-Abelian gauge theories has interesting nontrivial results. For one, in calculating quantum corrections to the classical conservation equation of *global axial flavor symmetries*, higher order Green functions (square and pentagon diagrams) must be taken into account giving the same result as

$$\partial_\mu j_5^{\mu,a} = -\frac{g^2}{16\pi^2} D^{abc} F^b \wedge F^c \equiv -\frac{g^2}{16\pi^2} D^{abc} \varepsilon^{\mu\nu\rho\lambda} F^b_{\mu\nu}(x) F^c_{\rho\lambda}(x), \tag{2.8}$$

where $D^{abc} \equiv \mathrm{tr}\left(t^a\{t^b,t^c\}\right)$; with t^b, t^c the generators of the non-Abelian gauge theory in the representation of the fermions circulating in the loop, and t^a the generator of the global flavor symmetry. It is these higher order diagrams that make (2.8) gauge invariant [18].[3]

For another, the *(chiral) local gauge symmetry current* which classically satisfies the covariant divergence equation

$$D_\mu J^\mu = 0, \tag{2.9}$$

now receives a quantum correction of the form

$$D^\mu J^a_\mu = -\frac{i}{12\pi^2}\varepsilon^{\mu\nu\rho\sigma}\mathrm{tr}\left(t^a\partial^\mu\left(A_\nu\partial_\rho A_\sigma - \frac{i}{2}A_\nu A_\rho A_\sigma\right)\right), \tag{2.10}$$

[3]Here, we have distinguished between the current $j^\mu_a = \bar{\psi}\gamma^\mu t_a\psi$, which plays the role of a *global flavor symmetry current* when t_a corresponds to *global* symmetries of the theory and a *non-Abelian gauge current* when t_a corresponds to the generator of the *local gauge* symmetry of the theory. Whereas the former satisfies ordinary classical conservation equation $\partial_\mu j^{\mu,5}_a = 0$ and is gauge invariant, the latter satisfies classically the covariant current conservation $D_\mu j^\mu_5 = 0$ and is gauge covariant.

satisfying the Wess-Zumino consistency conditions [2, 3, 4], where t^a is also a generator of the gauge group.

These quantum violations of symmetries occur for noncommutative gauge theories also, but with some surprises. For simplicity, we limit ourselves to the noncommutative version of massless QED. Noncommutative massless QED has the same Lagrangian as (2.2)

$$\mathcal{L} = \bar{\psi} \star (i\not\partial - g\not{A}) \star \psi - \frac{1}{4} F_{\mu\nu} \star F^{\mu\nu}, \tag{2.11}$$

with product of function replaced by $\star$-product

$$(f \star g)(x) \equiv f(x+\xi) \ \exp\left(\frac{i\Theta^{\mu\nu}}{2} \frac{\partial}{\partial\xi^\mu} \frac{\partial}{\partial\zeta^\nu}\right) g(x+\zeta)\Big|_{\xi=\zeta=0}, \tag{2.12}$$

e.g.

$$e^{ikx} \star e^{ipx} = e^{i(k+p)x} e^{ik\wedge p}, \tag{2.13}$$

with $k \wedge p = \Theta_{\mu\nu} k^\mu p^\nu$. From now on, we limit ourselves to the noncommutativity between space coordinates x_1 and x_2. Obviously $\star$-product is not a commutative operation and therefore noncommutative QED closely resembles an ordinary non-Abelian gauge theory. In fact in (2.11)

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]_\star. \tag{2.14}$$

Now noncommutative massless QED again has an axial $U(1)$ symmetry (2.1), $\psi \to e^{i\alpha\gamma_5}\psi$, to which now corresponds two currents

$$J^5_\mu \equiv \psi_\beta \star \bar{\psi}_\alpha (\gamma_\mu \gamma_5)^{\alpha\beta}, \tag{2.15}$$

and

$$j^5_\mu \equiv \bar{\psi}_\alpha \star \psi_\beta (\gamma_\mu \gamma_5)^{\alpha\beta}. \tag{2.16}$$

Under a $\star$-gauge transformation $\psi(x) \to e^{i\alpha(x)} \star \psi(x)$, these two currents are covariant and invariant, respectively. They have the same charge operator

$$Q_5 = \int d^3x j^0_5(x) = \int d^3x J^0_5(x). \tag{2.17}$$

Naively, one would expect the two currents to have the same anomaly. But, while

$$D_\mu J^\mu_5 = -\frac{g^2}{16\pi^2} F_{\mu\nu} \star \tilde{F}^{\mu\nu}, \tag{2.18}$$

the anomaly for the divergence of the "invariant" current j_5^μ is quite different. The reason for the difference of behavior of the two currents is an important property of noncommutative field theory, UV/IR mixing, which shows up in the "nonplanar" diagrams of the theory. Simply put, UV divergences of loop integrals, upon regularization, turn into IR divergences in the variable $\tilde{p}^\mu \equiv \Theta^{\mu\nu} p_\nu$.

In fact the first indication of nonvanishing anomaly in invariant current j_5^μ came from this source and emerges from the IR divergence of the triangle diagram integral [7]. Quoting the result from [7], the divergence of the invariant axial vector current is

$$
\begin{aligned}
\langle \partial_\mu j_5^\mu(x) \rangle = & -\frac{1}{\pi^2} \varepsilon_{\lambda\nu\alpha\beta} \int \frac{d^4k}{(2\pi)^4} \frac{d^4p}{(2\pi)^4}\; k^\alpha A^\lambda(k) e^{-ikx} p^\beta A^\nu(p) e^{-ipx} \\
& \times \int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_2\; \cos\left[k \wedge p(1 - 2\alpha_1 - 2\alpha_2)\right] \\
& \times \frac{1}{\ln \Lambda^2} \left\{ \left(\ln \frac{1}{\frac{1}{\Lambda^2} + \frac{q\circ q}{4}} - \ln \Delta \right) - \frac{q\circ q}{8} \left(\frac{1}{\frac{1}{\Lambda^2} + \frac{q\circ q}{4}} \right) - \Delta \ln \frac{1}{\frac{1}{\Lambda^2} + \frac{q\circ q}{4}} + \Delta \ln \Delta \right\}. \qquad (2.19)
\end{aligned}
$$

Here, $q \equiv k+p$, $\Delta = k^2\alpha_1(1-\alpha_1) + p^2\alpha_2(1-\alpha_2) + 2kp\,\alpha_1\alpha_2$, and $q \circ q \equiv -q_\mu \Theta^{\mu\nu}\Theta_{\nu\rho} q^\rho$. Λ is the cutoff regularization parameter. In the spirit of the UV/IR phenomenon [14], one considers the two different limits, $\frac{q\circ q}{4} \gg \frac{1}{\Lambda^2}$ and $\frac{q\circ q}{4} \ll \frac{1}{\Lambda^2}$, separately. In the first case, $\frac{q\circ q}{4} \gg \frac{1}{\Lambda^2}$, one takes the limit of the cutoff $\Lambda \to \infty$ keeping Θq finite. Then the anomaly vanishes due to the factor $\frac{1}{\ln \Lambda^2}$ in front of the expression on the third line of (2.19). On the other hand where $\frac{q\circ q}{4} \ll \frac{1}{\Lambda^2}$, a finite anomaly arises due to IR singularity; keeping Λ large but finite, *i.e.* considering the theory to be the low energy effective theory of a fundamental theory, and taking the limit $\Theta p \to 0$, a finite contribution from the factor

$$
\frac{1}{\ln \Lambda^2} \ln \frac{1}{\frac{1}{\Lambda^2}} = 1,
$$

in the third line of (2.19) gives the nonvanishing anomaly

$$
\langle \partial_\mu j_5^\mu(x) \rangle = -\frac{1}{16\pi^2} F_{\mu\nu} \star' \tilde{F}^{\mu\nu} + \cdots, \qquad (2.20)
$$

with the generalized $\star'$-product defined by

$$
f(x) \star' g(x) \equiv f(x) \frac{\sin\left(\frac{\Theta^{\mu\nu}}{2} \overleftarrow{\partial}_\mu \overrightarrow{\partial}_\nu \right)}{\frac{\Theta^{\mu\nu}}{2} \overleftarrow{\partial}_\mu \overrightarrow{\partial}_\nu} g(x). \qquad (2.21)
$$

Here, as it turns out the obtained result from the computation of triangle, square and pentagon diagram is not gauge invariant unless we add the contribution of infinitely many diagrams with

more and more external gauge field insertions. In [13] the nonplanar anomaly is calculated using the Fujikawa's path integral method and was argued that the expression on the r.h.s. of the above equation (2.20) is consistently gauge invariant only when $F_{\mu\nu}$ and $\tilde{F}^{\mu\nu}$ are attached to an open Wilson line as a function of the external gauge field. The ellipses on the r.h.s. of (2.21) indicate the contributions of the expansion of the Wilson line in the external gauge field.

Note that the expression above remains finite in low energy effective theory when the corresponding scale of the theory Λ is large comparing with $\frac{1}{|\Theta q|}$. However, considering the gauge theory as a fundamental field theory and letting Λ go to infinity, the integrand of the anomaly in the Fourier integral of (2.19) vanishes except for the point $q \circ q = 0$ for which the integrand becomes nonzero and finite in agreement with the point-splitting result of Ref. [10], which is then argued to be consistent with the constraint of Eq. (1.6).

But Fourier integral of a function which is everywhere zero and finite at a single point (not a Dirac δ-function!) is certainly zero which is the conclusion of zero anomaly of Ref. [9]. It is these conflicting conclusions that we set out to resolve in the next section. Our position is that because of UV/IR mixing of divergences in noncommutative theories, the necessarily nonvanishing anomaly of the invariant axial vector current will not emerge unless both UV and IR regularization are introduced in the calculation. The UV regularization is introduced by point-splitting the current; and the IR regularization is achieved by compactification of the noncommutative directions.

3 IR Regularized Invariant Anomaly

In this section we substantiate our claim that the resolution of the above paradox is by simultaneously regularizing the currents both in IR and UV regions. To calculate the anomaly of the divergence of the invariant axial current j_5^μ in the noncommutative QED we use the point-split, gauge invariant expression for the current similar to (2.7)

$$j_5^\mu(x) = \lim_{\epsilon \longrightarrow 0} \bar{\psi}_\alpha(x+\frac{\epsilon}{2}) \star \exp\left(-ig \int\limits_{x-\frac{\epsilon}{2}}^{x+\frac{\epsilon}{2}} A(y)\cdot dy\right)_\star \star (\gamma_\mu\gamma_5)^{\alpha\beta}\, \psi_\beta(x-\frac{\epsilon}{2}), \tag{3.1}$$

where the exponential is understood as $\star$-exponential. The ϵ-insertion is in fact a UV regulator which at the end of calculation goes to zero.

The divergence of the axial current in the limit of small ϵ then gives

$$\langle \partial_\mu j_5^\mu(x)\rangle = -ig \lim_{\epsilon\longrightarrow 0} \epsilon^\nu \langle \bar{\psi}_\alpha(x+\frac{\epsilon}{2}) \star F_{\mu\nu}(x) \star \left(\gamma^\mu\gamma^5\right)^{\alpha\beta} \psi_\beta(x+\frac{\epsilon}{2})\rangle. \tag{3.2}$$

Up to the first order of perturbation theory using the action (2.11), the above expression reads

$$
\begin{aligned}
&\langle \partial_\mu j_5^\mu(x)\rangle = \\
&= +ig^2 \lim_{\epsilon\to 0} \epsilon_\nu (\gamma_\mu\gamma_5)^{\alpha\beta}(\gamma_\rho)^{\sigma\tau} \Big\langle \bar\psi_\alpha(x+\frac{\epsilon}{2}) \overset{x}{\star} F^{\mu\nu}(x) \overset{x}{\star} \psi_\beta(x-\frac{\epsilon}{2}) \int d^4z\ \psi_\tau(z) \overset{z}{\star} \bar\psi_\sigma(z) \overset{z}{\star} A^\rho(z)\Big\rangle \\
&= -ig^2 \lim_{\epsilon\to 0} \epsilon_\nu \int d^4z\ \mathrm{tr}\left(S_F(z-x-\frac{\epsilon}{2})\gamma_\mu\gamma_5 \overset{x}{\star} F^{\mu\nu}(x) \overset{x\,z}{\star\star} S_F(x-z-\frac{\epsilon}{2})\gamma_\rho\right) \overset{z}{\star} A^\rho(z), \qquad (3.3)
\end{aligned}
$$

where we have indicated by overwrite the relevant $\star$-operators and further used the propagator of the massless fermions

$$
i\,(S_F(x-y))_{\alpha\beta} \equiv \langle \psi_\alpha(x)\bar\psi_\beta(y)\rangle = \frac{i}{(2\pi)^2}\,\frac{(x-y)_\mu\,(\gamma^\mu)_{\alpha\beta}}{\left[(x-y)^2-i\epsilon'\right]^2}. \qquad (3.4)
$$

Here, ϵ' arises from Feynman's ϵ-prescription. Now, it is known [8, 9] that the $\star$-operations render the integrals convergent and therefore as a result anomaly vanishes as $\epsilon \to 0$. It is also known that when the noncommutativity parameter Θ vanishes, the integral diverges and the usual commutative anomaly emerges. This indicates that under the circumstances that the $\star$-operation is inoperative a nonzero anomaly should be obtained. We will now make this statement precise.

To do so, we introduce an IR regulator by compactifying each space coordinates to a circle with radius R that plays the role of the IR regulator. The Fourier series expansion for the propagator (3.4) therefore reads

$$
S_F(z) = \sum_{\vec k}\int_{-\infty}^{+\infty} \frac{dk_0}{(2\pi)^{1/2}(2R)^{3/2}}\ \frac{\not k}{k^2}\ e^{-ik_0z_0}e^{+i\vec k\cdot\vec z}, \qquad \vec k \equiv \frac{\pi\vec n_k}{R}. \qquad (3.5)
$$

Using this expansion in (3.4) we get

$$
\begin{aligned}
\langle \partial_\mu j_5^\mu(x)\rangle &= -ig^2 \lim_{\epsilon\to 0}\epsilon_\nu \sum_{\vec p,\vec k}\int \frac{dp_0dq_0}{(2\pi)(2R)^3}\ \mathrm{tr}\left(\not q\gamma_\mu\gamma_5\not p\gamma_\rho\right) \\
&\times \int d^4z \frac{e^{-iq_0(z-x)_0}e^{+i\vec q\cdot(\vec z-\vec x-\vec\epsilon/2)}}{q^2} \overset{x}{\star} F^{\mu\nu}(x_0,\vec x) \overset{x\,z}{\star\star} \frac{e^{-ip_0(x-z)_0}e^{+i\vec p\cdot(\vec x-\vec z-\vec\epsilon/2)}}{p^2} \overset{z}{\star} A^\rho(z_0,\vec z). (3.6)
\end{aligned}
$$

The $\star$-products can be performed using

$$
e^{-i\vec p\cdot\vec z} \overset{z}{\star} f(z) \overset{z}{\star} e^{i\vec q\cdot\vec z} = f\left(z-(\widehat p+\widehat q)\right)\ e^{i\vec p\wedge\vec q}\ e^{-i(\vec p-\vec q)\cdot\vec z}, \qquad (3.7)
$$

with $\widehat p^i \equiv \frac{\Theta^{ij}p_j}{2}$, and $i,j=1,2$. After an appropriate change of variable and using $\mathrm{tr}\left(\gamma_\alpha\gamma_\mu\gamma_5\gamma_\beta\gamma_\rho\right) = 4i\varepsilon_{\beta\rho\alpha\mu}$, we obtain

$$
\begin{aligned}
\langle \partial_\mu j_5^\mu(x)\rangle &= -32g^2 \lim_{\epsilon\to 0}\epsilon_\nu\varepsilon_{\beta\rho\alpha\mu}\int d^4z \sum_{\vec k,\vec \ell}\int \frac{dk_0d\ell_0}{(2\pi)(2R)^3}e^{-i\ell(x-z)} \\
&\times \ell^\alpha A^\rho\left(z-\widehat k\right) F^{\mu\nu}\left(x-\widehat k\right) \frac{k^\beta e^{-i\vec k\cdot\vec\epsilon/2}}{(k+\ell)^2(k-\ell)^2}. \qquad (3.8)
\end{aligned}
$$

For finite noncommutative momentum $\widehat{\vec{k}}$, $F^{\mu\nu}$ and A^ρ will damp the k-integration/summation and a finite result will arise. Taking the zero limit of the UV regulator ϵ naively, would make the nonplanar anomaly vanish. But, as we will see, the nonplanar anomaly in fact receives a finite contribution from the zero modes of the product of A^ρ and $F^{\mu\nu}$. After performing a series expansion of these two fields and integrating over z, we get

$$\begin{aligned}\langle \partial_\mu j_5^\mu(x)\rangle &= -32g^2\varepsilon_{\beta\rho\alpha\mu}\sum_{\vec{\ell},\vec{s}}\int \frac{d\ell_0 ds_0}{(2\pi)(2R)^3}\ell^\alpha A^\rho(\ell)\, F^{\mu\nu}(s)\, e^{-i(s+\ell)x}\\ &\quad\times \lim_{\epsilon\to 0}\epsilon_\nu \sum_{\vec{k}}\int_{-\infty}^{+\infty} dk_0 \frac{k^\beta e^{-ik_i(\epsilon^i-\Theta^{ij}(\ell+s)_j)/2}}{(k+\ell)^2(k-\ell)^2}.\end{aligned} \tag{3.9}$$

Let us now concentrate on the integration/summation over k. Introducing the directions parallel and perpendicular to the noncommutative x_1-x_2 plane, $\vec{x}_\parallel = (x_0,x_3)$ and $\vec{x}_\perp = (x_1,x_2)$, we observe that the integral is in fact UV finite for $(\vec{\ell}+\vec{s})_\perp \neq \vec{0}$. In this case, the nonplanar anomaly vanishes after taking the zero limit of UV regulator ϵ. For vanishing $(\vec{\ell}+\vec{s})_\perp$, however, the integral

$$\mathcal{I}_i^{\ j} \equiv \lim_{\epsilon\to 0}\epsilon_i \sum_{\vec{k}}\int_{-\infty}^{+\infty} dk_0 \frac{k^j e^{-i\vec{k}\cdot\vec{\epsilon}/2}}{(k^2)^2} = 2i\lim_{\epsilon\to 0}\epsilon_i \frac{\partial}{\partial\epsilon_j}\sum_{\vec{k}}\int_{-\infty}^{+\infty} dk_0\ \frac{e^{-i\vec{k}\cdot\vec{\epsilon}/2}}{(k^2)^2}, \tag{3.10}$$

is the compactified version of the usual divergent integral leading to anomaly. Note that comparing with the expression on the last line of (3.9), we have neglected the ℓ-dependence in the denominator of (3.10). This is because we are only interested in the UV divergent part of this expression. In the limit of small ϵ, the discrete sum can be replaced by a logarithmical divergent integral leading to a finite result

$$\mathcal{I}_i^{\ j} \equiv iC\delta_i^{\ j}, \tag{3.11}$$

where $C = \frac{1}{(16\pi)^2}$ is a numerical factor. Before using this result a remark is in place:

As we have mentioned in the introduction, in [10] the non-compactified version of the same analysis is performed in the continuous momentum space $q = \ell + s$. The authors have argued that the nonplanar anomaly arises due to the UV divergence of the integral only when $\sum_{i,j=1,2}\Theta^{ij}(\ell+s)_j = 0$,[4] where in contrast to our case $\vec{q}_\perp \equiv (\vec{\ell}+\vec{s})_\perp$ is a *continuous* momentum variable. But, this is indeed a measure zero contribution to the Lebesgues integrand over q and

[4]In two dimensions $\Theta^{ij}(\ell+s)_j = 0$ would lead to $(\vec{\ell}+\vec{s})_j = 0$, with $j=1,2$.

does not contribute to the integral. Thus the nonplanar anomaly vanishes. In the compactified version, however, the zero mode is well-defined and makes a finite contribution to the anomaly.

Going back to our computation and inserting (3.11) together with a Kroeneker δ-function $\delta_{\vec{\ell}_\perp+\vec{s}_\perp,\vec{0}}$ in (3.9), we obtain

$$\langle \partial_\mu j_5^\mu(x)\rangle \;=\; -\frac{ig^2}{8\pi^2}\varepsilon_{j\rho\alpha\mu}\sum_{\vec{\ell},\vec{s}_\perp,s_3}\int \frac{d\ell_0 ds_0}{(2\pi)(2R)^3}\delta_{\vec{\ell}_\perp+\vec{s}_\perp,\vec{0}}\;\ell^\alpha A^\rho\left(\ell\right)F^{\mu j}\left(s\right)e^{-i(s+\ell)x}. \quad (3.12)$$

The Kroeneker δ-function indicates that the nonplanar anomaly exists only for vanishing discrete momentum $(\vec{\ell}+\vec{s})_\perp$. Performing further the sum over $\vec{s}_\perp$, the total $\vec{x}_\perp$ dependence of the resulting expression on the r.h.s. of (3.12) disappears and we are left with

$$\langle \partial_\mu j_5^\mu(x)\rangle = -\frac{ig^2}{8\pi^2}\varepsilon_{j\rho\alpha\mu}\sum_{\vec{\ell},s_3}\int \frac{d\ell_0 ds_0}{(2\pi)(2R)^3}\;\ell^\alpha A^\rho\left(\vec{\ell}_\parallel,\vec{\ell}_\perp\right)F^{\mu j}\left(\vec{s}_\parallel,-\vec{\ell}_\perp\right)e^{-i(\vec{s}_\parallel+\vec{\ell}_\parallel)\cdot\vec{x}_\parallel}. \quad (3.13)$$

Transforming the fields back to the coordinate space, we arrive at

$$\langle \partial_\mu j_5^\mu(x)\rangle = -\frac{g^2}{16\pi^2}\frac{1}{(2R)^2}\int_{-R}^{+R} d^2y_\perp\; F^{\alpha\rho}(\vec{x}_\parallel,\vec{y}_\perp)\tilde{F}_{\alpha\rho}(\vec{x}_\parallel,\vec{y}_\perp), \quad (3.14)$$

which can be interpreted as the zero modes in the Fourier expansion of the function $\mathcal{A}\equiv F\tilde{F}$ in the noncommutative coordinates $x_\perp$, *i.e.*,

$$\langle \partial_\mu j_5^\mu(x)\rangle = -\frac{g^2}{16\pi^2}\;\widetilde{\mathcal{A}}(x_\parallel,p_\perp=0), \quad (3.15)$$

the unintegrated form of the nonplanar anomaly. The result (3.14) and equivalently (3.15) are gauge invariant due to cyclicity of the $\star$-product under the integral over noncommutative coordinates $y_\perp$.

It is clear that an integration over $\vec{x}_\perp$ on both side of (3.14) cancels the R dependence on the r.h.s., and the integrated form of the nonplanar anomaly become

$$\int_{-R}^{+R} d^2x_\perp\langle \partial_\mu j_5^\mu(x)\rangle \;=\; -\frac{g^2}{16\pi^2}\;\frac{1}{(2R)^2}\int_{-R}^{+R} d^2x_\perp\int_{-R}^{+R} d^2y_\perp\; F^{\alpha\rho}(\vec{x}_\parallel,\vec{y}_\perp)\tilde{F}_{\alpha\rho}(\vec{x}_\parallel,\vec{y}_\perp), \quad (3.16)$$

which survives even in $R\to\infty$ limit, *i.e.*

$$\int_{-\infty}^{+\infty} d^2x_\perp\langle \partial_\mu j_5^\mu(x)\rangle = -\frac{g^2}{16\pi^2}\;\int_{-\infty}^{+\infty} d^2y_\perp\; F^{\alpha\rho}(\vec{x}_\parallel,\vec{y}_\perp)\tilde{F}_{\alpha\rho}(\vec{x}_\parallel,\vec{y}_\perp), \quad (3.17)$$

in agreement with the conclusion of [10].

Although this result coincides with the integrated form of nonplanar anomaly in [10], the unintegrated form of the anomaly (3.14) in the compactified version is in fact a novel result.[5] As we have seen the integration over noncommutative directions $\vec{x}_\perp$ should be performed before taking the limit $R \to \infty$ and this is the essential aspect of our IR regularization. Otherwise the nonplanar anomaly of the invariant current would vanish. And, the paradox pointed out in [10] would persist.

In the next section, we will use the same method to calculate the Schwinger terms in the current algebra of noncommutative currents.

4 Schwinger Terms

Anomalies as quantum breakdown of classical symmetries, appearing as contribution to the divergence of currents, are intimately related to another manifestation of quantum violation of these symmetries, *i.e.* in the additional terms to the current commutation relations of the symmetry of the theory, the so called Schwinger terms.

In fact axial anomalies were first observed in the context of the attempt to understand electroweak properties of hadrons through the study of the corresponding current algebras [3].

In the simplest case of massless QED with N_f flavors, the canonical algebra of currents of *global flavor symmetries*

$$[j_0^{5(a)}(\vec{x},t), j_0^b(\vec{y},t)] = if^{abc} j_0^{5(c)}(\vec{x},t)\delta^3(\vec{x}-\vec{y}), \tag{4.1}$$

receives a contribution due to quantum corrections proportional to derivatives of Dirac δ-function

$$[j_0^{5(a)}(\vec{x},t), j_0^b(\vec{y},t)] = if^{abc} j_0^{5(c)}(\vec{x},t)\delta^3(\vec{x}-\vec{y}) + c\delta^{ab}\varepsilon^{ijk}F_{jk}\partial_i\delta^3(\vec{x}-\vec{y}), \tag{4.2}$$

called Schwinger term. Here, the field strength tensor $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ and f^{abc} are the structure functions of the relevant groups, and the coefficient c is, what in the affine algebra context is called central charge. It is well-known that anomalies and Schwinger terms are

[5]There is an alternative calculation for the invariant anomaly [12] using Seiberg-Witten map and expansion of the currents in Θ, which yields an unintegrated form for the invariant anomaly. However, as convergence properties of diagrams depend crucially on considering all orders of the expansion, we hesitate to comment on their result. In fact nonplanar diagram integrals would have very different convergence properties order by order in Θ than the expression when the phases $e^{ip\wedge q}$ are not expanded in Θ.

related through the so called descent equations. However, a more direct connection between the equal-time commutation (ETC) relation and the anomaly is illustrated most easily in a detailed calculation of the anomaly in perturbation theory [see Appendix A].

In the case of noncommutative QED, as we have two distinct currents, the invariant currents j^μ or j_5^μ (2.15) and the covariant currents J^μ and J_5^μ (2.16), would have a number of different Schwinger terms. The Schwinger term appearing in the commutation relation of two covariant currents involves planar diagrams, whereas that corresponding commutator of a covariant and an invariant current involves nonplanar diagrams.

As an example[6], we calculate the equal-time commutation (ETC) relation between $J^{0,5}$ and J^0. It consists of a canonical and a Schwinger term. The canonical part is easily seen to be

$$\begin{aligned}\left[J_5^0(\vec{x},t), J^0(\vec{y},t))\right]\Big|_{\text{can.}} &= +2i\int \frac{d^4p}{(2\pi)^4}\,\frac{d^4q}{(2\pi)^4}\frac{d^4\ell}{(2\pi)^4}\;\psi_\beta(\ell+p+q)\bar{\psi}_\alpha(\ell)\,(\gamma_0\gamma_5)^{\alpha\beta}\\ &\quad\times e^{-i(p+q)_0 t}\; e^{+i\vec{p}\cdot\vec{x}}e^{+i\vec{q}\cdot\vec{y}}\; e^{-i\vec{\ell}\wedge(\vec{p}+\vec{q})}\;\sin\left(\vec{p}\wedge\vec{q}\right), \end{aligned} \tag{4.3}$$

which after some algebras becomes

$$\left[J_5^0(\vec{x},t), J^0(\vec{y},t))\right]\Big|_{\text{can.}} = \left[J_5^0(\vec{x},t) - J_5^0(\vec{y},t)\right]\star\delta^3\left(\vec{x}-\vec{y}\right), \tag{4.4}$$

and in momentum space

$$\left[J_5^0(\vec{p},t), J^0(\vec{q},t)\right]\Big|_{\text{can.}} = +2i\sin\left(\vec{p}\wedge\vec{q}\right)J_5^0\left(\vec{p}+\vec{q},t\right). \tag{4.5}$$

As for the corresponding Schwinger term, we have to calculate the vev of the ETC relation, which is related to the planar anomaly arising from the covariant current [6].[7] Using a point-splitting regularization and after some algebra we get

$$\begin{aligned}&\langle[J_5^0(\vec{x},t), J^0(\vec{y},t))]\rangle = \\ &= +32ig\lim_{\epsilon\to 0}\;\varepsilon_{\beta\rho\alpha 0}\int d^4z\int\frac{d^4\ell}{(2\pi)^4}\ell^\alpha A^\rho(z)e^{+i\ell_0(t-z_0)}\;e^{-i\vec{\ell}\cdot(\vec{x}-\vec{z})}\int\frac{d^4k}{(2\pi)^4}\;\frac{k^\beta\;e^{+i\vec{k}\cdot\vec{\epsilon}/2}}{(k+\ell)^2(k-\ell)^2}\\ &\quad\times\left(\delta^3(\vec{x}-\vec{y}+\frac{\vec{\epsilon}}{2}+\widehat{\vec{\ell}})-\delta^3(\vec{y}-\vec{x}+\frac{\vec{\epsilon}}{2}+\widehat{\vec{\ell}})\right),\end{aligned} \tag{4.6}$$

[6]The full algebra of noncommutative currents will be presented elsewhere [20].

[7]In Appendix A we have shown that in noncommutative QED, since the current appearing in the Lagrangian density is the covariant vector current J_μ, the calculation of the anomaly corresponding to the covariant axial vector current J_μ^5 involves the ETC of a covariant axial vector current with a covariant vector current, $[J_5^0(\vec{x},t), J^0(\vec{y},t)]$, whereas the calculation of the nonplanar anomaly corresponding to the invariant axial vector current j_μ^5 involves the ETC of an invariant axial vector current with a covariant current, $[j_5^0(\vec{x},t), J^0(\vec{y},t)]$.

at one-loop order, with $\widehat{\ell}$ defined by $\widehat{\ell}^i \equiv \frac{\Theta^{ij}\ell_j}{2}$. As we are interested in the UV behavior of the k-integration, we can neglect the ℓ dependence in the denominator, and arrive after some manipulations at

$$\begin{aligned} \langle [J_5^0(\vec{x},t), J^0(\vec{y},t))] \rangle &= -64ig \lim_{\epsilon\to 0} \varepsilon_{\beta\rho\alpha 0} \int d^4z\ \partial^\alpha A^\rho(z) e^{+i\ell_0(t-z_0)}\ e^{-i\vec{\ell}\cdot(\vec{x}-\vec{z})} \int \frac{d^4k}{(2\pi)^4} \frac{k^\beta\ e^{+i\vec{k}\cdot\vec{\epsilon}/2}}{(k^2)^2} \\ &\times \int \frac{d^3q}{(2\pi)^3}\ e^{+i\vec{q}\cdot(\vec{x}-\vec{y})} \left(\sin\left(\vec{q}\wedge\vec{\ell}\right) + \frac{\vec{\epsilon}\cdot\vec{q}}{2} \cos\left(\vec{q}\wedge\vec{\ell}\right) \right). \end{aligned} \tag{4.7}$$

The integral over k diverges linearly in the limit $\epsilon \to 0$. Taking an appropriate symmetric limit, the term proportional to $\sin\left(\vec{q}\wedge\vec{\ell}\right)$ cancels out and we are left with the term proportional to $\cos\left(\vec{q}\wedge\vec{\ell}\right)$. Using the result from previous section in performing the k-integration, we get

$$\begin{aligned} &\langle [J_5^0(\vec{x},t), J^0(\vec{y},t))] \rangle = \\ &= -\frac{g}{8\pi^2}\varepsilon^{ijk} \int d^4z\ \frac{d^4\ell}{(2\pi)^4}\ \frac{d^3q}{(2\pi)^3}\ \partial_k A_j(z) q_i\ e^{+i\ell_0(t-z_0)}\ e^{-i\vec{\ell}\cdot(\vec{x}-\vec{z})}\ e^{i\vec{q}\cdot(\vec{x}-\vec{y})}\ \cos(\vec{q}\wedge\vec{\ell}). \end{aligned}$$

The Schwinger term corresponding to the current algebra of the covariant currents then becomes

$$\langle [J_5^0(\vec{x},t), J^0(\vec{y},t))] \rangle = +\frac{ig}{16\pi^2}\varepsilon^{ijk} \Big[\partial_k A_j(\vec{x},t) \star \partial_i \delta^3(\vec{x}-\vec{y}) + \partial_i \delta^3(\vec{x}-\vec{y}) \star \partial_k A_j(\vec{x},t) \Big]. \tag{4.8}$$

In momentum space, it reads

$$\langle [J_5^0(\vec{p},t), J^0(\vec{q},t)] \rangle = -\frac{ig}{8\pi^2} \cos\left(\vec{p}\wedge\vec{q}\right) \varepsilon^{ijk} p_k q_i A_j\left(\vec{p}+\vec{q},t\right). \tag{4.9}$$

Combining the Schwinger term (4.8) [(4.9)] with the canonical term (4.4) [(4.5)], the ETC of covariant currents in the coordinate space is given by

$$\begin{aligned} [J_5^0(\vec{x},t), J^0(\vec{y},t))] &= \left(J_5^0(\vec{x},t) - J_5^0(\vec{y},t) \right) \star \delta^3(\vec{x}-\vec{y}) \\ &+ \frac{ig}{32\pi^2}\varepsilon^{ijk} \Big[F_{kj}(\vec{x},t) \star \partial_i \delta^3(\vec{x}-\vec{y}) + \partial_i \delta^3(\vec{x}-\vec{y}) \star F_{kj}(\vec{x},t) \Big], \end{aligned} \tag{4.10}$$

and in momentum space by

$$\begin{aligned} &[J_5^0(\vec{p},t), J^0(\vec{q},t)] = \\ &= +2i \sin\left(\vec{p}\wedge\vec{q}\right) J_5^0(\vec{p}+\vec{q},t) - \frac{ig}{8\pi^2} \cos\left(\vec{p}\wedge\vec{q}\right) \varepsilon^{ijk} p_k q_i A_j\left(\vec{p}+\vec{q},t\right). \end{aligned} \tag{4.11}$$

The ETC of covariant currents up to second order in Θ-expansion are calculated recently in [19] using an appropriate Seiberg-Witten map.

The Schwinger term of the commutator of an invariant current and a covariant current is related to the invariant current divergence and its anomaly, and as we have mentioned before involves nonplanar diagrams.

The ETC relation between the zero component of an invariant axial vector current j_5^0 and covariant vector current J^0

$$[j_5^0(\vec{x},t), J^0(\vec{y},t))],$$

involves only an anomalous term. This Schwinger term is calculated in a similar manner to the anomaly of section 3 using a point split regularization in three spatial directions. In the first order of perturbative expansion it reads

$$\begin{aligned}
&\langle [j_5^0(\vec{x},t), J^0(\vec{y},t))]\rangle = \\
&= +g\lim_{\epsilon\to 0}\Big[\delta^3(\vec{y}-\vec{x}+\frac{\vec{\epsilon}}{2}) \overset{x\,y}{\star\star}\, \psi_\beta(\vec{x}+\frac{\vec{\epsilon}}{2},t)\ \bar{\psi}_\alpha(y)\,(\gamma_0\gamma_5)^{\alpha\beta}\int d^4z\ \psi_\tau(z)\overset{z}{\star}\bar{\psi}_\sigma(z)\overset{z}{\star}(\gamma_\rho)^{\sigma\tau}A^\rho(z) \\
&\qquad -\psi_\beta(y)\ \bar{\psi}_\alpha(\vec{x}-\frac{\vec{\epsilon}}{2},t)\,(\gamma_0\gamma_5)^{\alpha\beta}\overset{x\,y}{\star\star}\,\delta^3(\vec{x}-\vec{y}+\frac{\vec{\epsilon}}{2})\int d^4z\ \psi_\tau(z)\overset{z}{\star}\bar{\psi}_\sigma(z)\overset{z}{\star}(\gamma_\rho)^{\sigma\tau}A^\rho(z)\Big]\Big|_{y_0=x_0} \\
&= +g\lim_{\epsilon\to 0}\Big[\int d^4z\ \delta^3(\vec{y}-\vec{x}+\frac{\vec{\epsilon}}{2})\overset{x\,y}{\star\star}\ \mathrm{tr}\left(S_F(z-y)\gamma_0\gamma_5\overset{z}{\star}S_F(\vec{x}-\vec{z}+\frac{\vec{\epsilon}}{2},t-z_0)\gamma_\rho\right)\overset{z}{\star}A^\rho(z) \\
&\qquad +\int d^4z\ \mathrm{tr}\left(S_F(\vec{z}-\vec{x}+\frac{\vec{\epsilon}}{2},z_0-t)\gamma_0\gamma_5\overset{z}{\star}S_F(y-z)\gamma_\rho\right)\overset{x\,y}{\star\star}\,\delta^3(\vec{x}-\vec{y}+\frac{\vec{\epsilon}}{2})\overset{z}{\star}A^\rho(z)\Big]\Big|_{y_0=x_0},
\end{aligned}$$

with the massless fermion propagator given in (3.4). As in the case of nonplanar anomaly, under certain circumstances the $\star$-product becomes inoperative and a nonvanishing Schwinger term emerges. To show this we perform an IR regularization by compactifying each space coordinate to a circle with radius R. After expanding the above result in the Fourier series and some straightforward manipulations we get

$$\begin{aligned}
&\langle [j_5^0(\vec{x},t), J^0(\vec{y},t))]\rangle = \\
&= +32ig\lim_{\epsilon\to 0}\varepsilon_{\beta\rho\alpha 0}\ \int d^4z\ \sum_{\vec{k},\vec{\ell}}\int\frac{dk_0d\ell_0}{(2\pi)(2R)^3}e^{+i\ell_0(t-z_0)}\ e^{-i\vec{\ell}\cdot(\vec{x}-\vec{z})}\ell^\alpha A^\rho\left(z-\widehat{k}\right)\frac{k^\beta e^{+i\vec{k}\cdot\vec{\epsilon}/2}}{(k+\ell)^2(k-\ell)^2} \\
&\quad\times\left(\delta^3(\vec{y}-\vec{x}+\frac{\vec{\epsilon}}{2}+\widehat{\vec{k}})-\delta^3(\vec{x}-\vec{y}+\frac{\vec{\epsilon}}{2}-\widehat{\vec{k}})\right). \qquad (4.12)
\end{aligned}$$

As expected this result is very similar to the result obtained for the nonplanar anomaly in the previous section [see Eq. (3.8)]. Here, as in the previous case, for finite $\widehat{\vec{k}}$ the integration/summation over k remains finite and the Schwinger term vanishes by taking naively the limit $\epsilon\to 0$. To show this, let us give the Schwinger term in the Fourier space,

$$\begin{aligned}
\langle [j_5^0(\vec{p},t), J^0(\vec{q},t)]\rangle &= -64g\ \epsilon_{\beta\rho\alpha 0}\sum_{\vec{\ell}}\int\frac{d\ell_0}{(2\pi)^{1/2}(2R)^{3/2}}e^{i\ell_0 t}\delta_{\vec{p}+\vec{q}+\vec{\ell},\vec{0}}\ \ell^\alpha A^\rho(-\ell) \\
&\quad\times\lim_{\epsilon\to 0}\ \sin\left(\frac{\vec{q}\cdot\vec{\epsilon}}{2}\right)\sum_{\vec{k}}\int_{-\infty}^{+\infty}dk_0\frac{k^\beta e^{+ik_i(\epsilon^i-\Theta^{ij}(q+\ell)_j)/2}}{(k+\ell)^2(k-\ell)^2}. \qquad (4.13)
\end{aligned}$$

As in the case of anomaly, the integral/sum over k is finite due to the damping effect of the gauge field A^ρ, as long as $(\vec{\ell}+\vec{q})_\perp$ is nonzero. In this case the Schwinger term vanishes in the limit $\epsilon \to 0$. For $(\vec{\ell}+\vec{q})_\perp = \vec{0}$, however, a finite contribution will arise. Its finite value will be calculated in the following. We expand the sin for small ϵ, and manipulate the k integration/summation as in (3.10). Using the result from (3.11), (4.13) reads

$$\begin{aligned}\langle[j_5^0(\vec{p},t), J^0(\vec{q},t)]\rangle &= +\frac{ig}{8\pi^2}\,\epsilon_{ijk}\sum_{\vec{\ell}}\int \frac{d\ell_0}{(2\pi)^{1/2}(2R)^{3/2}}e^{i\ell_0 t}\delta_{\vec{p}+\vec{q}+\vec{\ell},\vec{0}}\delta_{\vec{\ell}_\perp+\vec{q}_\perp,\vec{0}}\;q^i\ell^k A^j(-\ell),\\ &= -\frac{ig}{8\pi^2}\frac{1}{(2R)^{3/2}}\,\epsilon_{ij3}q^ip^3A^j(\vec{p}+\vec{q},t)\delta_{\vec{p}_\perp,\vec{0}},\end{aligned}\tag{4.14}$$

where the Kroeneker δ-function $\delta_{\vec{\ell}_\perp+\vec{q}_\perp,\vec{0}}$ is inserted to indicate that the Schwinger term exists only when $(\vec{\ell}+\vec{q})_\perp = \vec{0}$. Now going back to the coordinate space, and after some algebraic manipulations similar to the anomaly case, the first quantum correction to ETC of two different noncommutative currents in the coordinate space is given by

$$[j_5^0(\vec{x},t), J^0(\vec{y},t)] = +\frac{1}{(2R)^2}\,\frac{ig}{8\pi^2}\,\partial_3^x\delta(x_3-y_3)\varepsilon_{ij3}\partial^iA^j(\vec{y}_\perp,x_3,t),\tag{4.15}$$

and in momentum space,

$$[j_5^0(\vec{p},t), J^0(\vec{q},t)] = -\frac{ig}{8\pi^2}\,\frac{1}{(2R)^{3/2}}\,\epsilon_{ij3}q^ip^3A^j(\vec{p}+\vec{q},t)\delta_{\vec{p}_\perp,\vec{0}}.\tag{4.16}$$

Note that the R-dependence on the r.h.s. of (4.15) is also shared by the result (3.14) of the unintegrated form of the nonplanar (invariant) anomaly. In the discussion following (3.14) and leading to the integrated form of the anomaly from (3.17), we have shown that the R-dependence disappears upon integration over two noncommutative directions. In the case of current algebras, it is also possible to compare the results (4.11) for the ETC of two covariant currents and (4.15) for the ETC of an invariant and a covariant current to check the consistency of the result (4.15). To do this, it is enough to integrate these two relations over three spatial coordinates x and y, involving also the noncommutative directions and to show after taking the limit $R\to\infty$ that[8]

$$[Q_5^{inv.}, Q^{cov.}] = [Q_5^{cov.}, Q^{cov.}],$$

with $\qquad Q_5^{inv.} \equiv \int d^3x\; j_5^0(\vec{x},t), \qquad Q_5^{cov.} \equiv \int d^3x\; J_5^0(\vec{x},t),$

[8]Since the r.h.s. of (4.15) does not depend on noncommutative coordinates $\vec{x}_\perp = (x_1,x_2)$, its R-dependence is removed upon integration over these noncommutative coordinates.

and

$$Q^{cov.} \equiv \int d^3x \; J^0(\vec{x}, t).$$

As the case of commutator of two invariant currents is more involved and its physical significance less clear to us, we will postpone it to a more detailed publication [20].

5 Discussion

The lesson of UV/IR mixing in noncommutative field theories is that UV regularization of the theory does not render the theories consistent at the limit of zero momentum and the UV singularities reappear as IR singularities. In this work we have reexamined the invariant (nonplanar) anomaly and Schwinger terms in noncommutative $U(1)$ gauge theories with care. It became clear that the resolution of the question of nonzero integrated anomaly requires an IR cutoff, such as the compactification length. The anomaly and also the Schwinger term vanish as the compactification length is removed. This agrees with the observation of Intriligator and Kumar [9] that based on general properties of nonplanar diagram, argued that for finite noncommutativity parameter there is no UV divergence and hence no anomaly. These arguments were supported in analogy with string theory where the nonplanar anomalies vanish by Green-Schwarz (GS) mechanism of anomaly cancellation.

If we integrate the expression before removing the IR cutoff we obtain the integrated anomaly in agreement with the covariant anomaly. The result is very similar to the standard anomaly where it naively becomes zero if the UV divergence is not carefully handled. We also observe that the observation of Armoni, Lopez and Theisen [10] about nonzero result when $|\Theta p| = 0$ acquires meaning with finite IR cutoff where the Fourier integral becomes Fourier series and there is no zero measure difficulty. It is as if a finite charge is evenly distributed over the space giving zero density but still being totally nonzero. Therefore there is no ambiguity or inconsistency if the IR regulator is taken to its limit *after* the integration over noncommutative space coordinates are taken.

What is interesting is that the expressions for nonplanar anomalous term turn out to be independent of the noncommutative coordinates. Although we start with a local object, the divergence of a current, the result is an integral over the noncommutative part of the space washing all remnants of locality.

We can look at this phenomenon from two different points; pure field theory and String

theory. Looking from the field theory side the formula (3.8) shows that although we are considering divergence of the invariant current at a point, x, the expression involves points which are away by amount Θk. Upon integration over k this shift in position will cover the whole space without damping and makes the result independent of the position x where we are calculating the divergence. This is how large momentum integration which is a UV effect is reflected in the infinite wavelength, a constant term IR effect. Physically, large momentum states of the particle circulating in the loop are extended in the direction perpendicular to the momentum. This extension is the reason that the range of the nonlocality inherited from non commutativity extends to infinity, *i.e.* UV/IR mixing.

When the space is finite, this extension covers the compact dimension globally. This brings us to the stringy point of view. Such large extension reveals the stringy nature of noncommutative theories. The particle circulating in the loop is extended to the extent that it wraps around the compact direction and forms a winding state. The contribution of such winding states survives and leads to finite anomaly. Authors of Ref. [9] have suggested that anomaly cancels due to hidden GS-mechanism, *i.e.* via formation of closed string modes. This picture is complete only when the noncommutative plane is not compact. In the compact case, the extra winding states exist and couple to the states of the gauge theory. They develop IR poles proportional to R^2. As expected this contribution vanishes as the compactification length becomes infinitely large. At this limit the winding states decouple and their contribution to all processes including anomaly become zero.

Let us examine why this result can be interpreted as exchange of winding states. Winding states have masses proportional to R^2. Hence their contribution includes a propagator factor $1/((\Theta p)^2 - R^2)$ which becomes $1/R^2$ since the total noncommutative momentum is zero. This factor is present in the final result. The anomaly is constant in the noncommutative plane, which we attribute to the global nature of the winding states that are by nature nonlocal.

These observations explain why we need to resort to IR cutoff to extract the anomaly like the axial anomaly which needs careful UV regularization to be revealed. Similar points concerning contribution of winding states to nonplanar Feynman diagrams are discussed in the context of noncommutative field theories at nonzero temperature [21].

Winding states have similar manifestations in the Schwinger terms. In (4.15) we see that the anomalous contribution to the commutators of the invariant axial current and covariant current

becomes independent of the noncommutative coordinates of invariant axial current similar to the absence of noncommutative coordinates in invariant anomaly. Hence we conclude that winding string states are responsible for the Schwinger terms that also vanish at the infinite R limit supported by general theorems connecting the axial anomalies and Schwinger terms. We observe their validity also in the noncommutative case as is shown by direct calculations.

It is interesting to find how much of string theory is hidden in noncommutative theories and to what extent the spectrum and interactions of the underlying string theory can be extracted from them.

There are physical issues to be resolved concerning the decay of the Goldstone boson of the broken (global) symmetry. At first sight on can observe that the decay must be enhanced in the commutative directions. The details of the enhancement and its observable consequences are relegated to future publication. Since quantum effects are responsible for strong breaking of Lorentz invariance the meaning of the particle states have to be reexamined in the noncommutative cases which is needed for the interpretation of the results in particular the meaning of poles in the zero momentum in the noncommutative plane. We refer discussion on such consequences and other physical consequences to future publications.

6 Acknowledgments

This work is supported in part by Iranian TWAS chapter based at ISMO.

Appendix A

In this Appendix we will show the connection between the equal-time commutation (ETC) relation and the anomaly of the axial vector currents using perturbative expansion. In the ordinary commutative $U(1)$ gauge theory the action is given by

$$S \equiv \int d^4x \left[\bar{\psi}(x)(i\partial\!\!\!/ - m)\psi(x) - g\ j_\mu(x)A^\mu(x)\right], \tag{A.1}$$

where $j_\mu \equiv \bar{\psi}\gamma_\mu\psi$. Using this action, the v.e.v. of the divergence of the axial vector current $j_\mu^5 \equiv \bar{\psi}\gamma_\mu\gamma_5\psi$ is given in a perturbative expansion by

$$\langle\partial_\mu j_5^\mu(x)\rangle = -\frac{g^2}{2}\int d^4y\ d^4z\ \partial_\mu\Gamma^{\mu\nu\rho}(x,y,z)\ A_\nu(y)A_\rho(z), \tag{A.2}$$

with the vertex function

$$\Gamma_{\mu\nu\rho}(x,y,z) \equiv \langle T\left(j_\mu^5(x)j_\nu(y)j_\rho(z)\right)\rangle. \tag{A.3}$$

Building the divergence of $\Gamma_{\mu\nu\rho}(x,y,z)$ with respect to x, and using the definition of the T-ordered product, we arrive at

$$\begin{aligned}
\partial_x^\mu \Gamma_{\mu\nu\rho}(x,y,z) &= \partial_x^\mu \langle T\left(j_\mu^5(x) j_\nu(y) j_\rho(z)\right)\rangle \\
&= \langle T\left((\partial^\mu j_\mu^5(x))\ j_\nu(y) j_\rho(z)\right)\rangle + \delta(x^0-y^0)\langle T\left([j_0^5(x), j_\nu(y)]\ j_\rho(z)\right)\rangle \\
&\quad +\delta(x^0-z^0)\langle T\left(j_\nu(y)\ [j_0^5(x), j_\rho(z)]\right)\rangle. \qquad \text{(A.4)}
\end{aligned}$$

In the last two terms, there appears two ETC relations $[j_0^5(\vec{x},t), j_\nu(\vec{y},t)]$ and $[j_0^5(\vec{x},t), j_\rho(\vec{z},t)]$. This is a formal connection between the anomaly and the current algebra in the commutative $U(1)$-gauge theory.

In the following, we will use the same argument to show the connection between the anomaly corresponding to the covariant axial vector current J_μ^5 with the ETC of a covariant axial vector current and a covariant vector current, $[J_0^5(\vec{x},t), J_0(\vec{y},t)]$, and the nonplanar anomaly corresponding to the invariant axial vector current j_μ^5 with the ETC of an invariant axial vector current and a covariant current, $[J_0^5(\vec{x},t), J_0(\vec{y},t)]$.

The action of the noncommutative $U(1)$ gauge theory with fermions in the fundamental representation is given by

$$S_{\text{non-com}} \equiv \int d^4x \left[\bar{\psi}(x)(i\partial\!\!\!/ - m)\psi(x) - g\ J_\mu(x)\star A^\mu(x)\right], \qquad \text{(A.5)}$$

where $J_\mu \equiv \psi_\beta \star \bar{\psi}_\alpha(\gamma_\mu)^{\alpha\beta}$ is the covariant current. According to the above perturbative argument the divergence of the covariant axial vector current $J_\mu^5 \equiv \psi_\beta\star\bar{\psi}_\alpha(\gamma_\mu\gamma_5)^{\alpha\beta}$ is given by

$$\langle \partial_\mu J_5^\mu(x)\rangle = -\frac{g^2}{2}\int d^4y\ d^4z\ \partial_\mu \Gamma^{\mu\nu\rho}_{cov.}(x,y,z)\ A_\nu(y)A_\rho(z), \qquad \text{(A.6)}$$

where the vertex function of one *covariant* axial vector currents and two *covariant* vector currents

$$\Gamma^{cov.}_{\mu\nu\rho}(x,y,z) \equiv \langle T\left(J_\mu^5(x) J_\nu(y) J_\rho(z)\right)\rangle. \qquad \text{(A.7)}$$

Using again the definition of the T-ordered product, the partial derivation of $\Gamma^{cov.}_{\mu\nu\rho}$ with respect to x reads

$$\begin{aligned}
\partial_x^\mu \Gamma^{cov.}_{\mu\nu\rho}(x,y,z) &= \partial_x^\mu \langle T\left(J_\mu^5(x) J_\nu(y) J_\rho(z)\right)\rangle \\
&= \langle T\left((\partial^\mu J_\mu^5(x))\ J_\nu(y) J_\rho(z)\right)\rangle + \delta(x^0-y^0)\langle T\left([J_0^5(x), J_\nu(y)]\ J_\rho(z)\right)\rangle \\
&\quad +\delta(x^0-z^0)\langle T\left(J_\nu(y)\ [J_0^5(x), J_\rho(z)]\right)\rangle. \qquad \text{(A.8)}
\end{aligned}$$

On the r.h.s., there appears the ETC relations of a *covariant* axial vector current and a *covariant* vector current $[J_0^5(\vec{x},t), J_\nu(\vec{y},t)]$ and $[J_0^5(\vec{x},t), J_\rho(\vec{z},t)]$.

In contrast, by building the divergence of the invariant axial vector current $j_\mu^5 \equiv \bar{\psi}\ \gamma_\mu\gamma_5 \star \psi$, we arrive at

$$\langle \partial_\mu j_5^\mu(x)\rangle = -\frac{g^2}{2}\int d^4y\ d^4z\ \partial_\mu \Gamma_{inv.}^{\mu\nu\rho}(x,y,z)\ A_\nu(y)A_\rho(z), \tag{A.9}$$

with the vertex function of one *invariant* axial vector current and two *covariant* vector currents

$$\Gamma_{\mu\nu\rho}^{inv.}(x,y,z) \equiv \langle T\left(j_\mu^5(x)J_\nu(y)J_\rho(z)\right)\rangle. \tag{A.10}$$

The divergence of this vertex function involves the ETC of an *invariant* axial vector current and a *covariant* vector current $[j_0^5(\vec{x},t), J_\nu(\vec{y},t)]$ and $[j_0^5(\vec{x},t), J_\rho(\vec{z},t)]$

$$\begin{aligned}\partial_x^\mu \Gamma_{\mu\nu\rho}^{inv}(x,y,z) &= \partial_x^\mu \langle T\left(j_\mu^5(x)J_\nu(y)J_\rho(z)\right)\rangle \\ &= \langle T\left((\partial^\mu j_\mu^5(x))\ J_\nu(y)J_\rho(z)\right)\rangle + \delta(x^0-y^0)\langle T\left([j_0^5(x), J_\nu(y)]\ J_\rho(z)\right)\rangle \\ &\quad +\delta(x^0-z^0)\langle T\left(J_\nu(y)\ [j_0^5(x), J_\rho(z)]\right)\rangle. \end{aligned} \tag{A.11}$$

To find a more direct relation between the anomaly and the Schwinger terms, one has to calculate the full commutator algebra of noncommutative currents and insert it into the r.h.s. of the above equations (A.8) and (A.11). We will postpone this calculation to a more detailed publication [20].

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